13.3 Conditional Probability and Intersection of Events

Conditional probability is the probability of one event (F) happening assuming that another event (E) does.

Examples:

- probability that someone is happy given that they just won $$$.
- probability that someone passes an exam given that they did not study.

The probability that F happens given that E does is denoted $P(F|E)$

It is read “probability of F given E”

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{ TH, TT }

The event among those is that there is a Head.

{ TH }

P(H | 1st is T) = 1/2

Note that for the full experiment there are 4 outcomes, but we are only interested when the “given” outcome occurs.

Example: Roll a die for an experiment.

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The event assumed to happen was that the value was prime.

$\{ 2, 3, 5 \}$

Among those the event is when is it odd.

$\{ 3, 5 \}$

P(odd | prime) = 2/3

The previous examples lead to a way to count P(F | E) by a formula:

SPECIAL RULE FOR COMPUTING $P(F|E)$ BY COUNTING If E and F are events in a sample space with equally likely outcomes, then $P(F \mid E) = \dfrac{n(E \cap F)}{n(E)}$.

Recall: E ∩ F = E intersect F = E and F

Example: Two dice are rolled (order matters)

What is the probability that 1st die is 3 given that the sum is 4?

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Event “sum is 4”

Event “sum is 4 and 1^{st} die is 3”

Example: Two dice are rolled (order matters)

What is the probability that 1^{st} die is 3 given that the sum is 4?

Event "sum is 4"

$$\{\ (1, 3),\ (2, 2),\ (3, 1)\ \}$$

Event "sum is 4 and 1^{st} die is 3"

$$\{\ (3, 1)\ \}$$

$$P(1^{st} \text{ is } 3 \mid \text{sum is } 4) = \frac{n(1^{st} \text{ is 3 and sum is } 4)}{n(\text{sum is } 4)}$$

$$= 1/3$$

Conditional Probability

- Example: Assume that we roll two dice and the total showing is greater than nine. What is the probability that the total is odd?

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Conditional Probability

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- Solution: This sample space has 36 equally likely outcomes. We will let *G* be the event "we roll a total greater than nine" and let *O* be the event "the total is odd." Therefore,

$$G = \{(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}.$$

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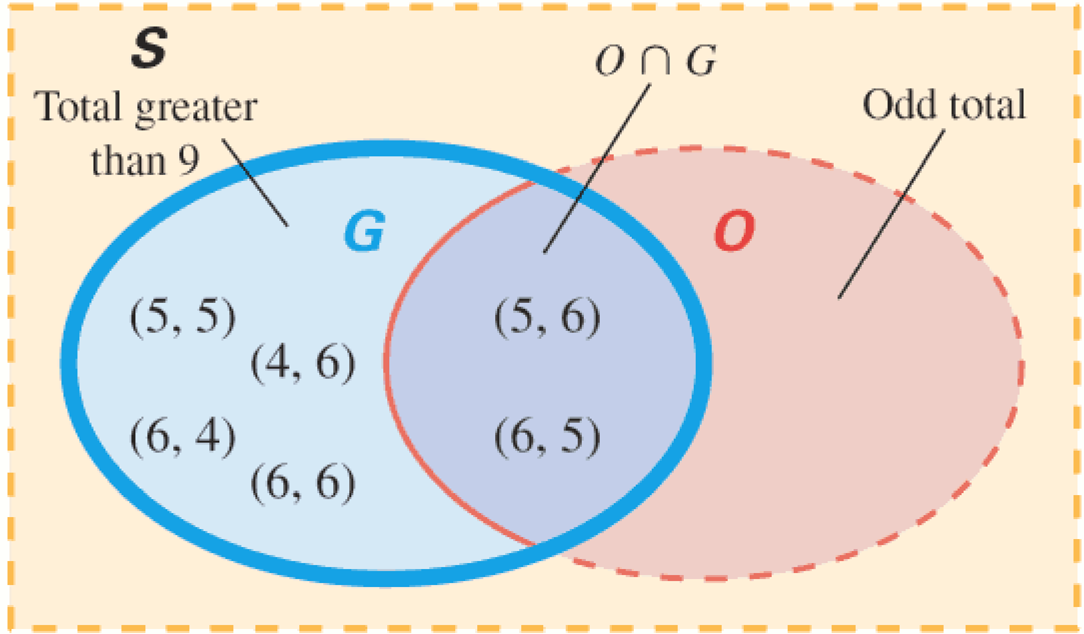
Conditional Probability

We now seek all pairs that give an odd total – the diagram below shows that there are two.



$$P(O|G) = \frac{n(O \cap G)}{n(G)} = \frac{2}{6} = \frac{1}{3}$$

Note that $P(E \mid F)$ and $P(F \mid E)$ are different.

Example:

If $n(E) = 4$, $n(F) = 8$, and
$n(E \cap F) = 2$

$$P(E \mid F) = P(E \cap F) / P(F)$$
$$= 2/4 = 1/2$$

$$P (F \mid E) = P(E \cap F) / P(E)$$
$$= 2/8 = 1/4$$

By multiplying through on the formula...

RULE FOR COMPUTING THE PROBABILITY OF THE INTERSECTION OF EVENTS If E and F are two events, then

$$P(E \cap F) = P(E) \cdot P(F \mid E).$$

In testing for a disease, a test works 90% of the time given that the person has the disease.
10% of the people have the disease.

What is the probability that someone has the disease and the test works?

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$P(\text{test works} \mid \text{disease}) = 0.9$
$P(\text{disease}) = 0.1$

$$
\begin{aligned}
P(\text{test works and disease}) &= P(\text{test works} \mid \text{disease}) \times P(\text{disease}) \\
&= 0.9 \times 0.1 \\
&= 0.09
\end{aligned}
$$

The Intersection of Events

- Example: Assume your professor has written questions on 10 assigned readings on cards and you are to randomly select two cards and write an essay on them. If you have read 8 of the 10 readings, what is your probability of getting two questions that you can answer?

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The Intersection of Events

- Example: Assume your professor has written questions on 10 assigned readings on cards and you are to randomly select two cards and write an essay on them. If you have read 8 of the 10 readings, what is your probability of getting two questions that you can answer?
- Solution: Let *A* be "you can answer the first question;" and *B* be "you can answer the second question."

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The Intersection of Events

We need to calculate

probability you can answer the first question

probability you can answer the second question, given that you answered the first question

$$P(A \cap B) = \underbrace{P(A)}_{\text{probability you can answer the first question}} \cdot \underbrace{P(B \mid A)}_{\text{probability you can answer the second question, given that you answered the first question}}.$$

We may compute the following probabilities:

$$P(A) = \frac{8}{10} \qquad P(B \mid A) = \frac{7}{9}$$

$$P(A \cap B) = P(A) \cdot P(B \mid A) = \frac{8}{10} \cdot \frac{7}{9} = \frac{56}{90} \approx 0.62$$